Closing Thu: 15.3, 15.4
Midterm 2 is Tuesday, March 1
It covers 13.3/4, 14.1/3/4/7, 15.1-15.4
Closing Next Thu: 15.5

Recall a few additional double integral applications

$$\iint_R 1\, dA = \text{Area of R}$$

$$\frac{1}{\text{Area of R}} \iint_R f(x,y) dA$$
$$= \text{Average value of f(x, y) over R}$$

Entry Task:
The temperature at each point on the xy-plane is given by T(x,y) = 3xsin(y) degrees Celsius.
Find the average temperature over the region R on the xy-plane bounded by y = 0, $y = x^2$, and x = 4.

15.4 Double Integrals over Polar Regions

Recall:

θ = angle measured from positive x-axis

r = distance from origin

$x = r\cos(\theta)$, $y = r\sin(\theta)$, $x^2 + y^2 = r^2$

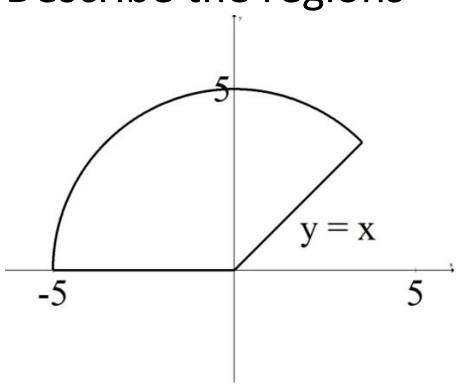
To set up a double integral in polar we will:

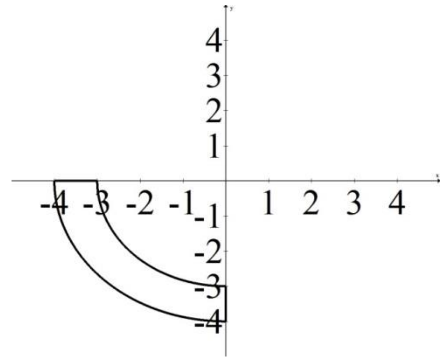
1. Describing the region in polar
2. Replace "x" by "$r\cos(\theta)$"
3. Replace "y" by "$r\sin(\theta)$"
4. Replace "dA" by "$r\,dr\,d\theta$"

Step 1: Describing regions in polar.

Examples:

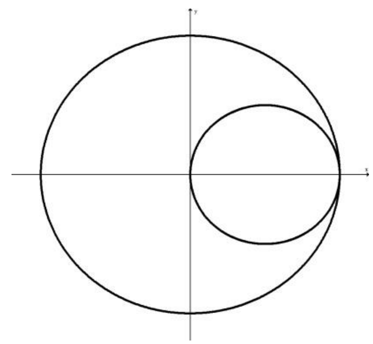
Describe the regions





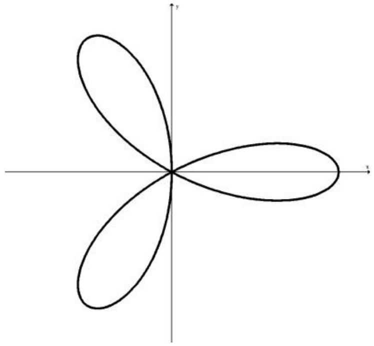
Some homework

HW 15.4/4: Describe the region in the first quadrant between the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 4x$ using polar.



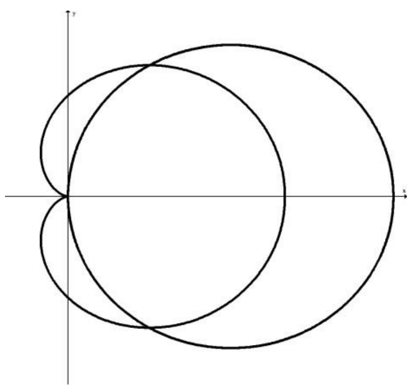
HW 15.4/5:

Describe one closed loop of $r = 6\cos(3\theta)$.



HW 15.4/7:

Describe the region inside $r = 1+\cos(\theta)$
and outside $r = 3\cos(\theta)$.



General Note About Ch. 15:

If given a description of a solid in words,

1. Solve for "z" anywhere you see it. That is your integrand(s).
2. Graph region in the xy-plane.
   - Graph all given x and y constraints.
   - Find intersection of all surfaces.

Examples:

HW 15.3/10:

Find the volume enclosed by $z = 4x^2 + 4y^2$ and the planes $x = 0$, $y = 2$, $y = x$, and $z = 0$.

HW 15.4/8:

Find the volume below $z = 18 - 2x^2 - 2y^2$ and above the xy-plane.

HW 15.4/9:

Find the volume enclosed by $-x^2 - y^2 + z^2 = 22$ and $z = 5$.

HW 15.4/10:

Find the volume above the upper cone $z = \sqrt{x^2 + y^2}$ and below $x^2 + y^2 + z^2 = 81$

Step 2: Set up your integral in polar.

Conceptual notes:

Cartesian

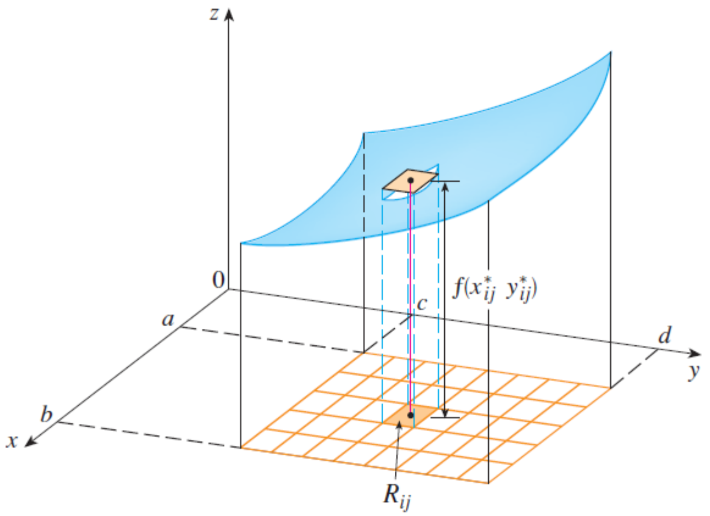
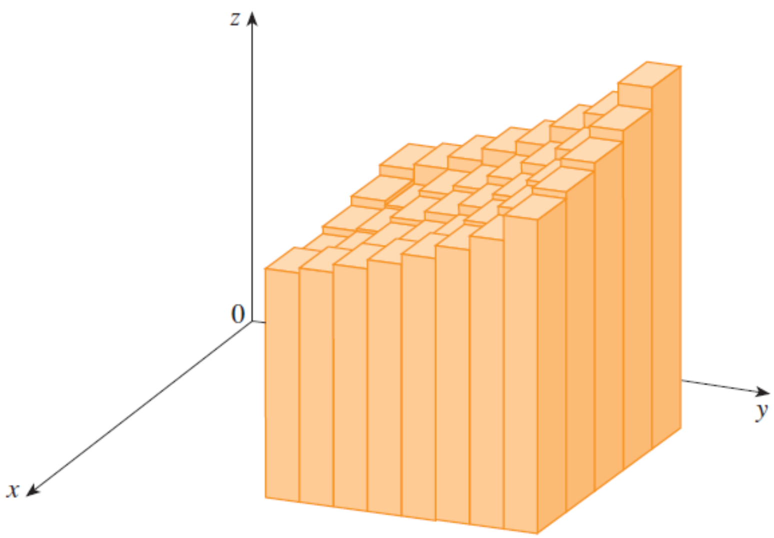
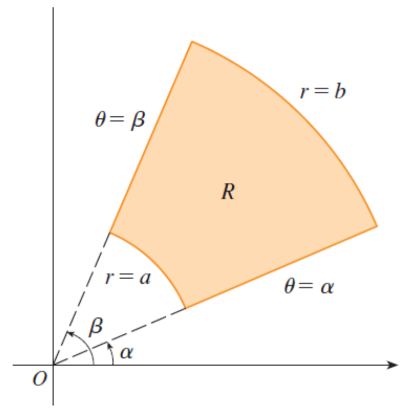
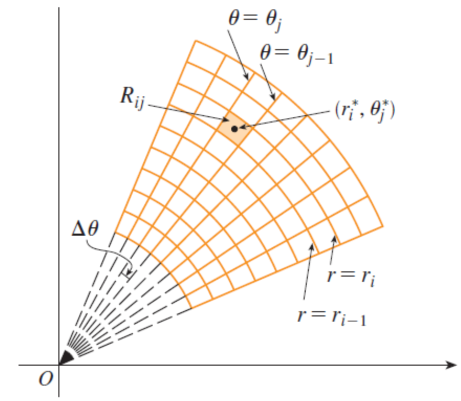


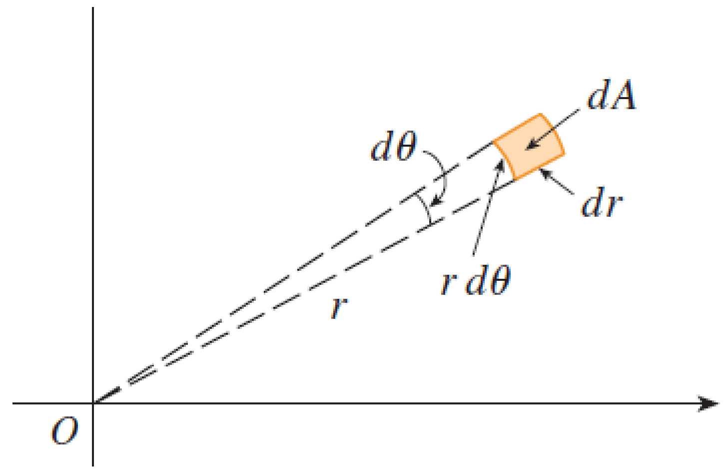
FIGURE 4



Polar







Examples:

1. Compute

$$\iint_R \frac{\cos(\sqrt{x^2+y^2})}{\sqrt{x^2+y^2}}\,dA$$

2. **HW 15.4/5**:

Find the area of one closed loop of r = 6cos(3θ).

3. **HW 15.4/4**:

Evaluate

$$\iint_R x\,dA$$

over the region in the first quadrant between the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 4x$ using polar

Moral:
Three ways to set up a double integral:
"Top/Bottom":

$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

"Left/Right":

$$\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

"Inside/Outside":

$$\iint_R f(x,y)\,dA$$

$$= \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r\cos(\theta), r\sin(\theta))\,r\,dr\,d\theta$$